Give **very** brief answers. Explanations are **not** required. **SCORE: _____ / __2__ PTS**

[1] If $f(x) \geq g(x)$ for all x, and $\int_0^\infty f(x)\,dx$ converges, what can you conclude about $\int_0^\infty g(x)\,dx$?

NO CONCLUSION ①

[2] If $x^{-3} \leq h(x)$ for all x, what can you conclude about $\int_1^\infty h(x)\,dx$?

NO CONCLUSION ①

MULTIPLE CHOICE Circle the correct answers. **SCORE: _____ / __4__ PTS**

[1] Consider the three integrals #1: $\int_0^{\frac{\pi}{4}} \csc x\,dx$ #2: $\int_{-1}^{1} \frac{1}{x^2-4}\,dx$ #3: $\int_1^e \ln(x-1)\,dx$

DISCONT @ $x=0$

DISCONT @ $x=1$

Which of the integrals above are improper ?

[A] none [B] only #1 ② [C] only #2 [D] only #3

[E] only #1 and #2 [✓ [F] only #1 and #3] [G] only #2 and #3 [H] all

[2] Consider the three integrals #1: $\int_0^\infty (0.9)^x\,dx$ #2: $\int_2^\infty \frac{1}{x^\pi}\,dx$ #3: $\int_0^5 \frac{1}{x^e}\,dx$

$0<0.9<1$

$\pi>1$

Which of the integrals above converge ?

[A] none ② [B] only #1 [C] only #2 [D] only #3

[✓ [E] only #1 and #2] [F] only #1 and #3 [G] only #2 and #3 [H] all

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Evaluate $\int \frac{t-12}{t^3+4t^2+4t}\,dt$. **SCORE: _____ / __7__ PTS**

$$\frac{t-12}{t(t+2)^2} = \frac{A}{t} + \frac{B}{t+2} + \frac{C}{(t+2)^2} = \frac{A(t+2)^2 + Bt(t+2) + Ct}{t(t+2)^2}$$

$$\underline{t-12 = A(t+2)^2 + Bt(t+2) + Ct} \ ①$$

$t=0: \ -12 = 4A \rightarrow A = -3$

$t=-2: \ -14 = -2C \rightarrow C = 7$

COEF OF t^2: $0 = A + B \rightarrow B = -A = 3$

$$\int\left(\overset{①}{\boxed{\frac{-3}{t}}} + \overset{①}{\boxed{\frac{3}{t+2}}} + \overset{①}{\boxed{\frac{7}{(t+2)^2}}}\right)dt$$

$$= \underbrace{-3\ln|t|}_{\textcircled{\tfrac{1}{2}}} + \underbrace{3\ln|t+2|}_{①} \underbrace{- \frac{7}{t+2}}_{①} \underbrace{+C}_{\textcircled{\tfrac{1}{2}}}$$

SANITY CHECK

$t=2 \quad \frac{-10}{32} = \frac{-5}{16} \quad \frac{-3}{2} + \frac{3}{4} + \frac{7}{16} = \frac{-24+12+7}{16} = \frac{-5}{16}$ ✓

Evaluate $\int \ln(x^2 - 6x + 13)\, dx$. **SCORE:** _____ / 7 **PTS**

$u = \ln(x^2-6x+13) \quad dv = dx$

$du = \frac{2x-6}{x^2-6x+13}\,dx \quad v = x$

$$\begin{array}{r} 2 \\ x^2-6x+13 \overline{) 2x^2-6x} \\ \underline{2x^2-12x+26} \\ 6x-26 \end{array}$$

$\frac{6x-26}{x^2-6x+13} = \frac{A(2x-6)+B}{x^2-6x+13}$

$6x-26 = A(2x-6)+B$

$x=3: \; -8 = B$

COEF OF x: $6 = 2A \to A = 3$

$\underbrace{x\ln(x^2-6x+13)}_{①} - \underbrace{\int \frac{2x^2-6x}{x^2-6x+13}\,dx}$ ①

$= x\ln(x^2-6x+13) - \int \Big(2 + \underbrace{\frac{6x-26}{x^2-6x+13}}_{①}\Big)dx$

$= x\ln(x^2-6x+13) - 2x - \int \boxed{\frac{3(2x-6)-8}{x^2-6x+13}}\,dx$ ①

$= x\ln(x^2-6x+13) - 2x - 3\ln(x^2-6x+13)$

$\qquad + 8\int \frac{1}{(x-3)^2+2^2}\,dx$

(½) ①

$= x\ln(x^2-6x+13) \overbrace{-2x} \overbrace{- 3\ln(x^2-6x+13)}$

$\qquad \underbrace{+ 4\tan^{-1}\frac{x-3}{2}}_{①} \underbrace{+ C}_{(½)}$

Evaluate $\int \frac{dr}{1+\cos r}$. **SCORE:** _____ / 4 **PTS**

$= \int \boxed{\frac{1-\cos r}{(1+\cos r)(1-\cos r)}}\,dr$ ①

$= \int \boxed{\frac{1-\cos r}{\sin^2 r}}\,dr$ (½)

$= \int \underbrace{(\csc^2 r - \cot r \csc r)}_{①}\,dr$

$= \underbrace{-\cot r + \csc r}_{①} \underbrace{+ C}_{(½)}$

Evaluate $\int_1^\infty \frac{1}{x\ln x}\,dx$. If the integral diverges, write "DIVERGES". **SCORE:** _____ / 6 **PTS**

$\int \frac{1}{x\ln x}\,dx \quad u = \ln x$

$= \int \frac{1}{u}\,du = \ln|u| = \underbrace{\ln|\ln x|}$ ①

$= \underbrace{\int_1^e \frac{1}{x\ln x}\,dx + \int_e^\infty \frac{1}{x\ln x}\,dx}_{①}$

$\underbrace{\text{DIVERGES}}$ ①

NOTE:
YOU DO NOT HAVE TO
USE e SPECIFICLY;
ANY NUMBER GREATER
THAN 1 CAN BE USED
IN PLACE OF e

$\int_1^e \frac{1}{x\ln x}\,dx = \lim_{N\to 1^+} \ln|\ln x|\Big|_N^e$

(GRADE AGAINST 1 VERSION ONLY) ↳ EITHER ONE OK

$= \underbrace{\boxed{\lim_{N\to 1^+}} -\ln|\ln N|}$ DNE ① ①

$(\ln N \to 0^+, \underbrace{\ln|\ln N| \to -\infty}_{①})$

OR

$\int_e^\infty \frac{1}{x\ln x}\,dx = \lim_{M\to\infty} \ln|\ln x|\Big|_e^M$

$= \underbrace{\boxed{\lim_{M\to\infty}} \ln|\ln M|}$ DNE ① ①

$(\ln M \to \infty, \underbrace{\ln|\ln M| \to \infty}_{①})$